Model Predictive Control with Embedded Reference Dynamics for precise trajectory tracking in an underactuated two-link multibody system

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EXTENDED ABSTRACT

1 Introduction

Accurate control of underactuated multibody systems (i.e., when the number of independent control inputs is less than the number of degrees of freedom) is a challenging issue in several robotic applications, such as cranes, robots with passive joints, flexible joint or flexible links [1]. In the last years a lot of effort has been devoted, within the multibody community, to develop effective controllers for fast and effective trajectory tracking control. Basically, these methods can be grouped into feedforward and feedback control. Feedforward is an open loop strategy aimed at computing the time history of the control forces allowing the controlled system to track the desired reference, usually by exploiting a detailed system model to be inverted; feedback, in contrast, exploits the measurements of a proper set of sensors to compute such control forces and to reject disturbances, especially when applied to the unactuated coordinates. On the one hand, feedforward techniques are usually not enough to ensure good tracking performances because of the presence of model uncertainty and external disturbances: hence, they are usually implemented together with feedback ones. On the other hand, feedback control alone might be not satisfactory for tracking time-varying reference trajectories with high frequency harmonic components, especially those beyond the speed loop bandwidth.

The difficulties in controlling underactuated multibody systems are exacerbated in the case of non-minimum phase systems, i.e., with unstable internal dynamics, for both feedback and feedforward control. First, if the system features an odd number of real right-half plane zeros in the transfer function (of the linearized dynamics in case of nonlinear systems) from the input force to controlled position, undershoot is experienced in tracking the reference trajectory. Moreover, if a time-varying reference trajectory is specified, tracking delay and errors become relevant. Secondly, model inversion is challenging and requires pre and post actuation, or approximated (i.e., stabilized) solutions [2] .

In this paper, precise trajectory tracking in underactuated non-minimum phase multibody systems is solved by adopting an improved formulation of Model Predictive Control (MPC). The idea of standard MPC is to solve a constrained optimal control problem over a receding horizon to compute the optimal sequence of the control input. MPC was originally applied to control power plants and chemical processes, and lately it has been applied in regulation problems in motion control, where one or more outputs are controlled to assume reference values in a finite time after a step change of the reference. A critical issue of standard MPC schemes is that the reference signal is kept constant during the prediction horizon, thus leading to a piecewise-constant approximation of a time varying trajectory; therefore, the reference is usually tracked with a lag.

To overcome these difficulties, that increase in the case of underactuated non-minimum phase systems, an improved formulation proposed by the Authors in [3] is adopted in this work to ensure precise, and with negligible lag, tracking of a time-varying reference commanded for the tip of a two-link system. Additionally, thanks to the embedding of the reference, no feedforward is needed, thus overcoming the difficulties in solving the inverse dynamics problem for this kind of systems.

2 Brief description of the MPC-ERD

The proposed approach is, therefore named, MPC-ERD, MPC with Embedded Reference Dynamics, and it is just briefly outlined in this Extended Abstract. The underlying idea of the MPC-ERD is to embed an autonomous state-space model of the time-varying reference, over the prediction horizon, in the optimization to be solved for computing the sequence of optimal control actions. To obtain the following autonomous state-space representation of the reference $\mathbf{r}(k)$ (k is the discrete time),

$$\begin{cases} \mathbf{x}_\mathrm{r}(k+1) = \mathbf{A}_\mathrm{r}\,\mathbf{x}_\mathrm{r}(k) \\ \mathbf{r}(k) = \mathbf{C}_\mathrm{r}\,\mathbf{x}_\mathrm{r}(k) \end{cases} \tag{1}$$

this paper exploits the theory of Dynamic Mode Decomposition (DMD) [4] to model arbitrary references through the dynamic matrix $\mathbf{A}_\mathrm{r}$, the output matrix $\mathbf{C}_\mathrm{r}$, and an "internal" state vector $\mathbf{x}_\mathrm{r}(k)$. By including the dynamics of both the reference and of the multibody system to be controlled (the latter by means of a linearized dynamic model formulated as a state-space model), the MPC-ERD is capable of tracking with almost-zero lag and transient tracking error the time-varying reference. To improve tracking performances, an embedded integrator is introduced by exploiting the difference variables, leading to the so-called "velocity form of MPC". Constraints on the motor torque are also included in the optimization problem used to compute the control action, to ensure feasibility and avoid unexpected control saturations.

3 Test case

The method is numerically applied to a planar two-link robot arm with two revolute joints, as shown in Figure 1, that is often used to mimic a flexible arm, that is supposed to move a payload on the tip. Joint A is actuated by a motor, while joint B is passive with a torsional spring. The system has two degrees of freedom (θ_1, θ_2 in Figure 1) and just one driving torque, τ_m, is adopted to control the motion. By choosing the tip translation in the X direction as the controlled output, x_{tip}, then the system is non-minimum phase [3] , thus making the control design more challenging and providing a severe test for the proposed approach. Only a sample result is shown in this Abstract: the method is general and can handle other trajectories. To show a meaningful application, a desired reference recalling a pick-and-place is defined from the pick location to the place one, by connecting the two points through a 5th-degree polynomial motion law. In the simulation, the system is supposed to be equipped by two encoders with 4000 pulses, to measure θ_1, θ_2, and therefore speed is estimated through filtered numerical derivatives.

The results in Figure 1 clearly shows the capability of the proposed approach to provide precise and fast tracking of the time-varying reference, with just a small error (the peak is equal to 1.2 mm, while the root mean square value is 0.6 mm) and zero delay. Additionally, the small computational effort of the proposed scheme allows its real time implementation.

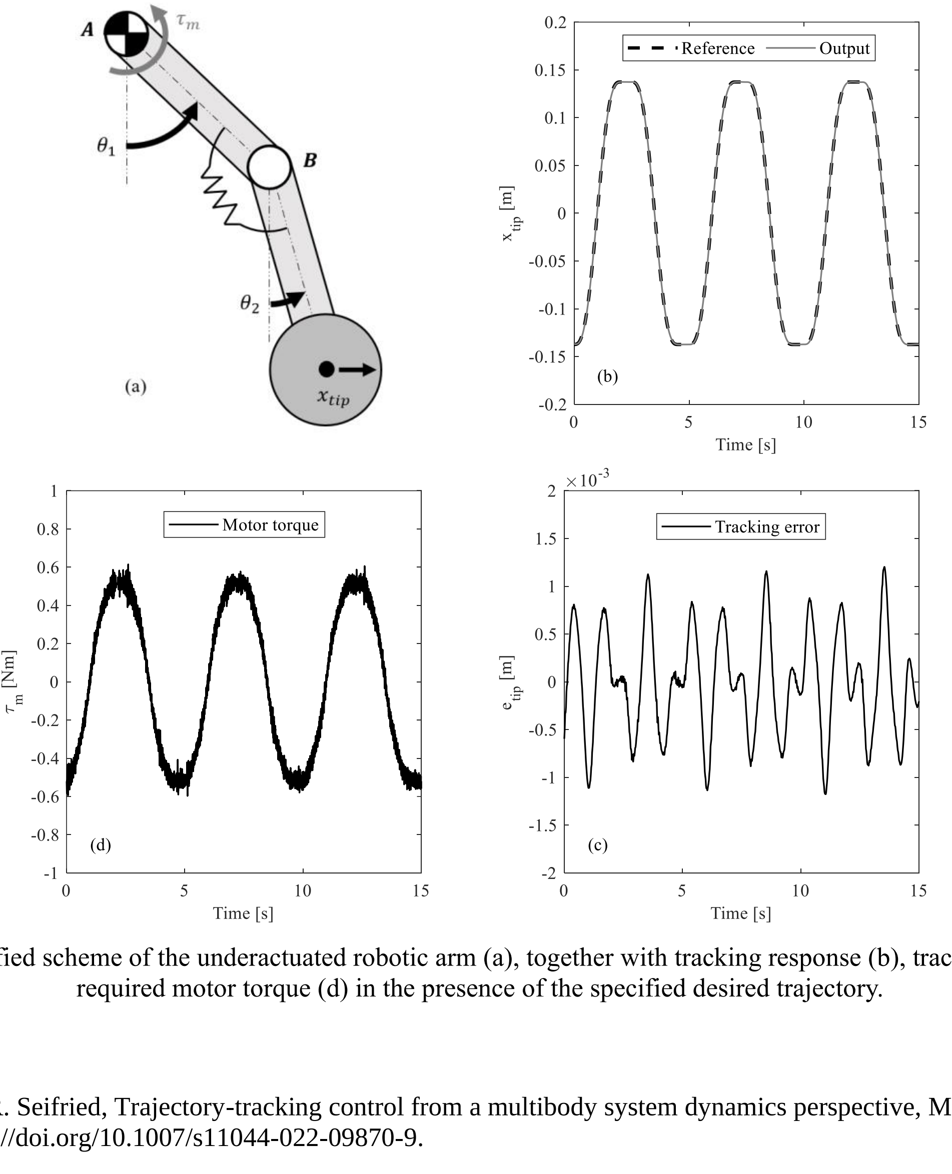

Figure 1: Simplified scheme of the underactuated robotic arm (a), together with tracking response (b), tracking error (c) and required motor torque (d) in the presence of the specified desired trajectory.

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